## When Will We Ever Use This? <br> Benchmaking 101

Mr. Sam Ligo<br>Instructor, Austin Peay State University<br>Department of Mathematics and Statistics ligos@apsu.edu

## Purpose

- Our purpose today is to act as students who have been asked to build a $1: 4$ scale model of a real park bench.
- Through the process, we will be exercising several math and geometry skills as well as critical thinking skills.
- This can be done in groups in your classroom. It does require a bit of preparation and a reasonable expense, but a kit can last for years.
- If you are making one kit, making two kits takes only a little bit longer. So, make four or five at once.



## Equipment

- The kit
- 12 ft of 1 x 2 pine board (I use 3 six-foot sticks)
- Paint if desired (easier removal of glue)
- Miter saw with adjustable angles
- Chunk of $2 \times 4$ pine board
- Activity Day
- Wood Kit
- Calculator
- Hot glue gun
- Protractors (paper/plastic/both)
- Rulers
- Scraper and hair dryer to remove glue



## Skills Exercised

- Following basic, specific directions
- Critical thinking
- Scaling measurements up and down
- Reading a fractional scale
- Fractions and decimal equivalents
- Transferring measurements
- Calculating and using supplementary angles
- Calculating and using complementary angles
- Identifying various geometric shapes
- Projecting lines to form hypothetical shapes
- Using properties of triangles

Transitioning from 2D to 3D representation (nets)

## Caveats

- Measurements on the actual wood pieces are to the nearest eighth of an inch
- Please be conservative with the amount of glue used; it doesn't take much.
- This bench that we are building is not a true scaled replica of the actual bench. This is because the wood is accurately scaled on length but not on width and thickness.
- Carpenters measure angles differently than we do in the classroom, so I have converted everything to classroom techniques for the exercise.


## How we will measure angles this morning

- A carpenter would call this a $20^{\circ}$ angle; measured on the short axis of the board


- A student would call this a $70^{\circ}$ angle; measured on the long axis of the board.


## How we measure angles this morning



- A carpenter would call this a $20^{\circ}$ angle
- A student would call this a $70^{\circ}$ angle
- I will be using the student version for all angle discussions today; measured on the long axis.


## Warm up Exercise

- Measure the diameter of the Oreo in inches.
- Scale it up using a $1: 4$ scale
- How big is the scaled up Oreo?



## Critical Thinking

- As we scale models up and down, the length, width, and height of the structural members will change.
- Will the angles change? How is a 3-4-5 triangle like a 6-8-10 triangle?



## When is a $2 \times 4$ not a $2 \times 4$ ?



## When is a $2 \times 4$ not a $2 \times 4$ ?

- Measure the actual width and thickness of your piece of 2 x 4
- The $4 "$ width is actually $\qquad$ 99
- The 2 " thickness is actually $\qquad$ , 9
- Using a 1:4 scale, the width of the model wood should be $\qquad$ " but it is
$\qquad$ ".
- Using a $1: 4$ scale, the thickness of the model wood should be $\qquad$ " but it is

$\qquad$


## Accountability and Grouping

- How many pieces of $1 \times 2$ pine do you have?
- Group all of your pieces into piles of identical pieces. How many piles do you have?
- Notice that one piece in each group is lettered with A through F.


## Find

- Find the piece that would be 47 " long on the real bench. How long is the scaled down piece? $\qquad$
- What letter is it? $\qquad$
- Mark the length of the scaled down piece on this image

- Mark the piece of the scaled up piece on this image



## Find

- Find the piece that looks like a parallelogram. What letter is it? $\qquad$
- Is it actually a parallelogram? Why or why not? What are the two angles?
$\qquad$ and $\qquad$
- How long was this scaled down "piece before the angles were nipped off of each end? $\qquad$ "
- Mark the length of the scaled down piece on this image

- Mark the length of the scaled up piece on this image



## Equate



- Mark 26 3/8 (1)
- Mark 26 4/8 (2)
- Mark 26 4/16 (3)
- Mark 26 15/16 (4)

Mark 27.0625 (5)
Mark 27.25
(6)

Mark 27.875 (7)
Mark 27.6875 (8)

## Find

- Find the piece whose end angles are $90^{\circ}$ and $62.5^{\circ}$. What letter is it? $\qquad$
- Find the piece whose supplementary angle is $95^{\circ}$. What letter is it? $\qquad$
- Find the piece whose complementary angle is $12.5^{\circ}$. What letter is it? $\qquad$ -


## Find

- Find the piece whose end angles are $90^{\circ}$ and $62.5^{\circ}$. What letter is it? $\qquad$ D
- Find the piece whose supplementary angle is $95^{\circ}$. What letter is it? $\qquad$ F
- Find the piece whose complementary angel is $12.5^{\circ}$. What letter is it? $\qquad$ B


## Let's begin construction: Left Side

- Use two C pieces top and bottom. The two C pieces are not parallel.
- Put the F piece on the right and the E piece on the left as shown.
- Notice the blue lines on E and F.



## Left Side continued

- Now, add a D piece as shown.
- Align the D piece with the two blue lines on E and F .
- Ensure that D is perpendicular with F .
- Ensure the ends of D are perfectly flush with the sides of E and F .
- Ensure that the top of D is parallel with and adjacent to both blue lines on E and F
- Glue D onto E and F



## Add Left Back Support

- Glue B on top of E with the angled end of B abutted against the top of $D$.
- Ensure that B and E are forming a $140^{\circ}$ angle.
- Ensure that the angled end of $B$ is properly aligned with the intersection of E and D.



## Right Side

- Use two C pieces top and bottom. The two C pieces are not parallel.
- Put the F piece on the left and the E piece on the right as shown.



## Right Side continued

- Now, add a D piece as shown.
- Align the D piece with the two blue lines on E and F .
- Ensure that D is perpendicular with F.
- Ensure the ends of D are perfectly flush with the sides of E and F .
- Ensure that D is parallel with and adjacent to both blue lines on E and $F$
- Glue D onto E and F



## Add Right Back Support

- Glue B on top of E with the angled end of B abutted against the top of $D$.
- Ensure that B and E are forming a $140^{\circ}$ angle.
- Ensure that the angled end of $B$ is properly aligned with the intersection of E and D.



## Add Slats for Back and Seat

- Glue the frontmost C onto the seat.
- Glue the upper most C onto the back. Ensure the ends of $C$ are flush with the sides and ends of B.



## Add More Slats

- Using the $1 / 4$ " spacer between the C pieces, add the next C to the back and the next C to the seat.
- Add the third C to the back of the bench.



## Add the Arms (L and R)

- Glue A to the tops of E and F.
- Ensure that the back of A is flush with the back of B.



## And now we have a bench!



## Parallel Members?

- Is the seat parallel to the ground? Long axis? Short axis?
- Are the arms parallel the seat?
- Are the arms perpendicular to the back?
- Is the seat perpendicular to the back?



## Height?

- How high is your model?
- How high is the real bench?



## Consider the partial triangle formed by the legs with the ground




- Find the bottom two angles with your protractor.
- Solve for the top angle.
- Measure the bottom of the triangle on your bench in inches.
- Empirically solve for the two unknown sides using a ruler and your personal scale on your handout.
- Consider using cm to make these measurements.



## What are the bottom angles?






## Inclinometer resting on the arm



## In closing...

- Thank you for your attendance and participation
- Thank you for what you do every day for our young people
- My business card are available. Drop me an email if you'd like a copy of these slides and your mini-bench plans
- Best wishes for a productive spring semester
- I will be raffling off these bench sets if you wish to participate please give me your ticket stub.

